Exercise 3

Find the angle between $7\mathbf{j} + 19\mathbf{k}$ and $-2\mathbf{i} - \mathbf{j}$ (to the nearest degree).

Solution

Use the definition of the dot product to obtain the angle θ between these vectors.

$$
\begin{aligned}
(7\mathbf{j} + 19\mathbf{k}) \cdot (-2\mathbf{i} - \mathbf{j}) &= \|7\mathbf{j} + 19\mathbf{k}\|\|-2\mathbf{i} - \mathbf{j}\| \cos\theta \\
&= \sqrt{7^2 + 19^2}\sqrt{(-2)^2 + (-1)^2} \cos\theta \\
&= \sqrt{410}\sqrt{5}\cos\theta
\end{aligned}
$$

Solve for $\cos\theta$.

$$
\begin{aligned}
\cos\theta &= \frac{(7\mathbf{j} + 19\mathbf{k}) \cdot (-2\mathbf{i} - \mathbf{j})}{\sqrt{410}\sqrt{5}} \\
&= \frac{(0)(-2) + (7)(-1) + (19)(0)}{\sqrt{410}\sqrt{5}} \\
&= \frac{-7}{\sqrt{410}\sqrt{5}}
\end{aligned}
$$

Therefore,

$$
\theta = \cos^{-1}\left(\frac{-7}{\sqrt{410}\sqrt{5}}\right) \approx 99°.
$$